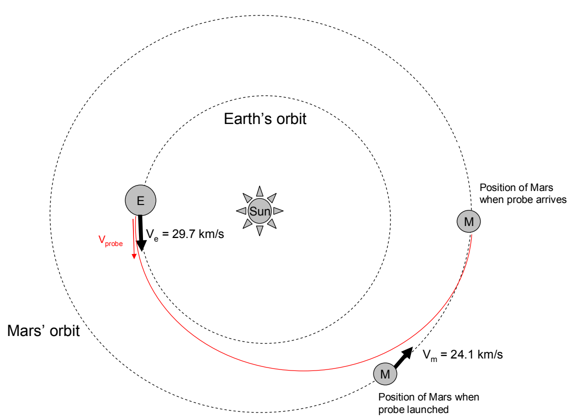
Interplanetary Trajectories

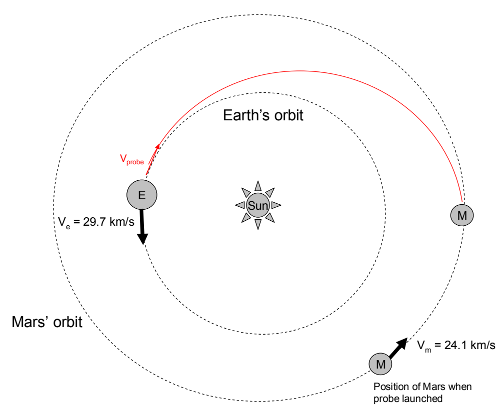
Interplanetary trajectories build on the basic properties of orbits. In the article on Kepler's laws we saw how basic orbits take the shape of conic sections with the focus at the centre of the planet. In the section on orbital manoeuvre we saw how it is possible to move between orbits by simple velocity changes. Interplanetary manoeuvres exploit and combine these basic ideas.

In this section we'll start by considering a trivial example, and use this to identify some of the difficulties of trajectory design. After this we'll look at a more realistic design method. Finally, we'll look at the oft misunderstood slingshot manoeuvre.

A SIMPLE EXAMPLE

Let's start with a simple trajectory, a transit from Earth to Mars. We'll time our launch so that, at the end of the transit, Mars is at a point in its orbit directly opposite the point at which the Earth was when the probe was launched. The trajectory approximates to a Hoffmann transfer with the Sun at one focus of the ellipse. There are two possible directions for the launch: in the direction of travel of the earth (figure 1) and in the opposite direction (figure 2). When the probe is launched in the direction of travel of the Earth it gains an extra 29.7 km/s due to the motion of the earth, thus the total delta-v of the launch is reduced by this amount. Less fuel is required at launch when the direction of travel is the same as the Earth.





Mars is travelling around the Sun at approximately 24.1 km/s. In order to intercept Mars at the correct point the probe has to be launched when Mars lags the intercept pint by a distance corresponding to the flight time.

This trivial example treats the interplanetary trajectory as a simple Hoffman transfer. In practice a trajectory based on such a simple model would miss Mars for several reasons:

1. The trajectory would be inaccurate. The times and velocities need precise calculation as as even a small error would accumulate into a very significant error over the long flight times.
2. The gravity of other planets will affect the trajectory, notably the earth in the early stages of the flight and mars in the latter stages. The gas giants, particularly Jupiter, also exert a perturbing force on the trajectory.
3. The orbits of the planets in the solar system are not perfectly circular but are elliptical, albeit with small eccentricity. This complicates the trajectory.

Point 1 is self explanatory, and dictates the need for high accuracy in the trajectory design. As launches and burns are imperfect there is always some initial error in the trajectory. One way of mitigating these errors is to allow a fuel margin onboard the probe for a mid course correction. How much fuel to add is a matter of judgement: too much fuel increases the probes weight and a compensating reduction in payload weight is required – never popular with the scientists!

Point 2 can be managed by a good design method. A mathematical analysis of all the individual gravitational forces on a probe is complex. It is known as the "n-body problem" as there are "n" bodies (planets and the Sun), and each body exerts a force on the probe. As all the bodies are in relative motion the resulting force vector is continuously changing. An analytical solution to the n-body problem is mathematically hideous, so numerical methods are usually employed.

Numerical methods set start point of the planets and the probe all the forces on the probe are calculated as if the planets are stationary. A computer then calculates the resulting motion of the probe due to all the gravitational forces for a small time interval, perhaps the next few seconds of flight. The position of all the planets and the probe is then re-plotted and the calculations repeated. By repeating these calculations many millions of times the whole trajectory of the probe can be calculated and a "final" position of the probe determined.

The resolution of Point 3 is fairly straightforward with numerical analysis as the instantaneous position of the planets can be calculated.

A REALISTIC METHOD

The problem with numerical methods is that a long computer run will only tell you where the probe ends up. It is very difficult to tune the variables to produce a realistic trajectory that links a start time and point to an end time and point via a minimal set of manoeuvres. The art of this method is to know which variables to correct (launch time, arrival time, velocities, masses etc) to arrive at a satisfactory trajectory. Some supporting method is clearly required that will allow the trajectory designer top get a simple approximation to the variables, and then fine-tune this to arrive at a useable trajectory. Such a technique exists, and it is called the "patched conic approximation" or PCA.

The PCA "patches" different segments of a trajectory into a sequence. Let's revisit our simple example of a space probe transiting from Earth orbit to Mars orbit, this time giving the planets arbitrary start points. At the start of this sequence of manoeuvres the space probe is in a circular Earth orbit. Earth and Mars are at different points in their orbit around the Sun. The relative positions of the bodies are shown in Figure 3. The probes orbit and subsequent trajectory are shown in red.

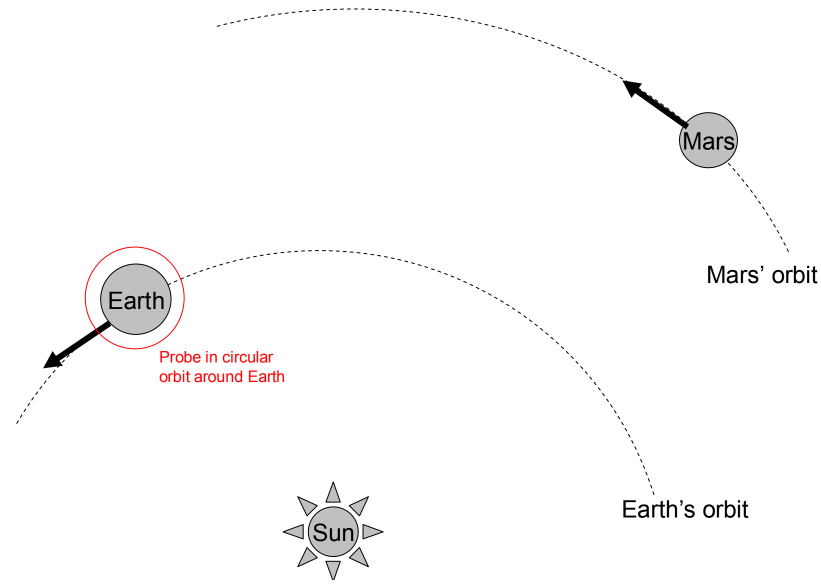


Figure 3- Initial Positions

In the first manoeuvre we conduct a burn to inject the probe into an orbit that will allow it to intersect the orbit of Mars. The manoeuvre is shown in Figure 4.

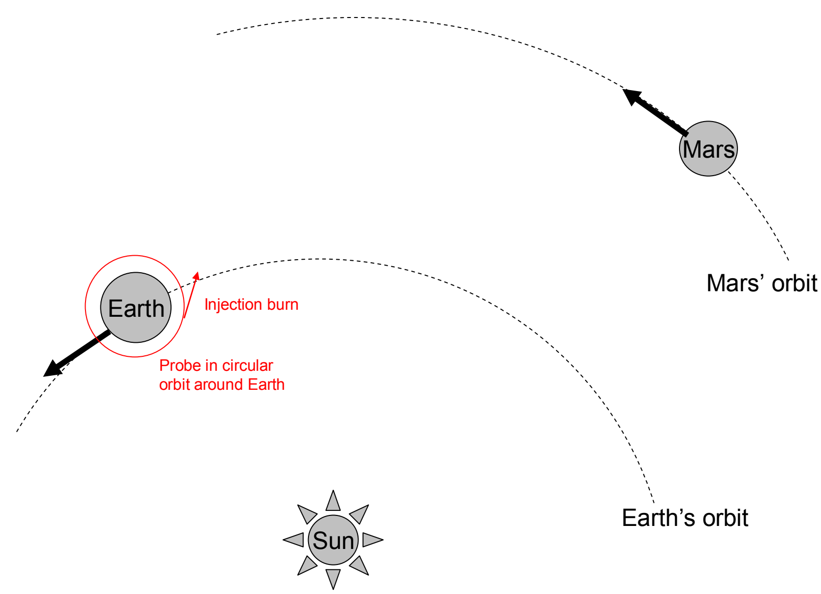


Figure 4 – Injection burn

Initially the probe moves under Earth's gravity in an orbit that permits it to intercept the orbit of Mars. At some point it will leave the gravitational pull of the Earth and its trajectory will be dominated by the gravity of the Sun. For simplicity the PCA defines an imaginary spherical surface around the Earth. Within this "sphere of influence" (SOI) we only consider the effects of Earth's gravity and outside it we only consider the Sun's gravity. This is slightly artificial as the transition from Earth's to the Sun's gravitational fields is gradual; however it greatly simplifies the mathematics. We show this point at which the trajectory leaves the Earth's SOI as point 1 in Figure 5.

We can calculate the radius of a planet's SOI very simply:

$$R_{SOI} = R_{SP} \left(\frac{M_P}{M_S} \right)^{2/5}$$

Where:
R_{SOI} = radius of the sphere of influence
R_{SP} = radius of the planets orbit around the Sun
M_P = mass of the planet
M_S = mass of the Sun

Using this equation we can find the SOI for planets with an approximately circular orbit. The R_{SOI} for the Earth is approximately 927,000 km, or about slightly under 150 Earth radii.

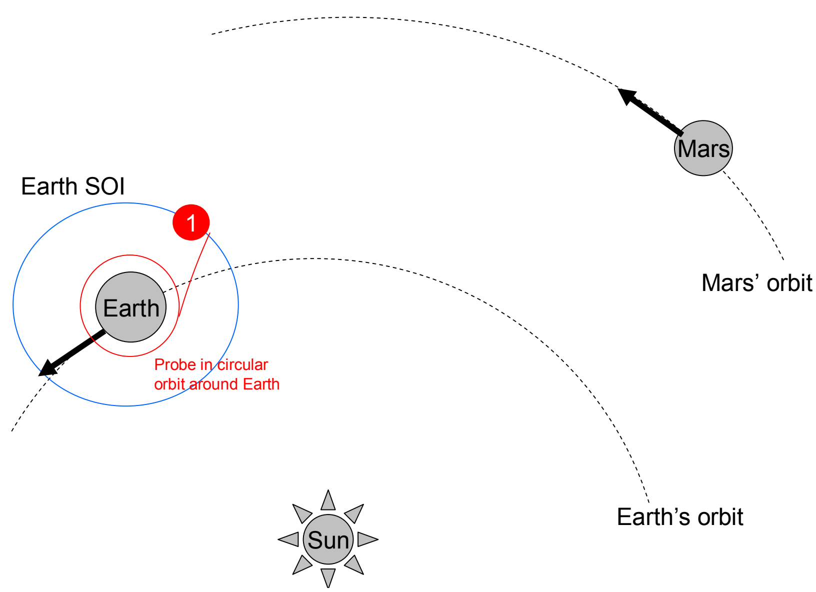


Figure 5 – Leaving Earth's SOI

The probe is now subject only to the forces of gravity from the Sun and follows an interplanetary trajectory. This will be an ellipse, parabola or hyperbola with the Sun at one focus. The shape of the orbit will depend on the probes velocity, distance from the

Sun and μ_{Sun} as described in the section on Keplers laws. Eventually the probe will enter the SOI of Mars as shown in Figure 6

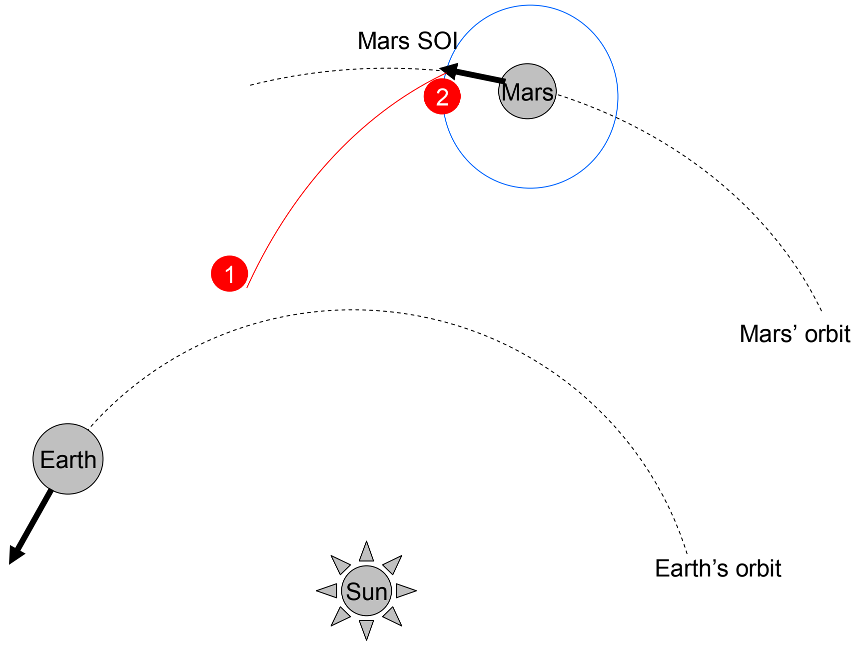


Figure 6 – Interplanetary trajectory

Once inside the SOI of Mars the probe will be subject only to the gravity of Mars and will execute an orbit with Mars at the Focus. Left unattended it will perform a fly-by, leaving Mars SOI at the same velocity (relative to Mars) as it arrived. There are a lot of misconceptions about this manoeuvre, which is called a "slingshot" in popular science, and it will be analyzed in more depth later on.

In our trajectory we need to enter a circular orbit around Mars so we decelerate the probe. This is normally done be rotating the probe by 180 degrees in either pitch or yaw, and doing a burn to provide some negative delta-v. More recent Mars probes have conserved fuel by using the Martian atmosphere to aerobrake. This takes longer than a burn, but tends to produce a good circular orbit over a period of time. This last manoeuvre can be seen in Figure 7.

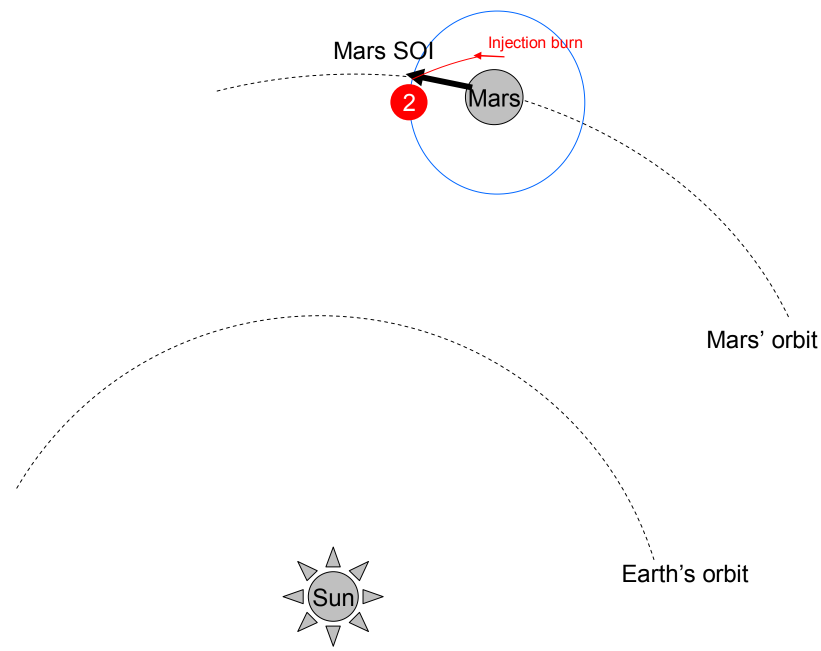


Figure 7 – Mars orbit injection burn

In this analysis we've basically treated each leg of the trajectory as a restricted 2-body problem. The restriction is that one of the bodies (the probe) is very much lighter than the other bodies (the Earth, the Sun or Mars). As a result the gravitation attractions will deflect the path of the lighter body but will not deflect the heavier body. Each leg has thus been a simple conic section, and it has allowed us to calculate the velocity vector (speed and direction) at which each leg ends and the next leg begins. You can see why the method is called the patched conic approximation:

- Consecutive legs of the trajectory are **patched** together in sequence
- Each leg is a simple **conic** section as it is a solution to the restricted 2-body problem
- The restriction of the problem to two bodies simplifies the maths and gives a fair **approximation** to the actual trajectory.

This method has allowed us to produce an approximation to the trajectory, and gives strong pointers to the values of the variables in a proper numerical analysis.

<u>THE GRAVITATIONAL SLINGSHOT</u>

There's a lot of misunderstanding about the slingshot manoeuvre. We need to go back to basics to understand the slingshot and where the additional velocity comes from.

Imagine a probe approaching Mars on a fly-by trajectory. As the probe enters the SOI of Mars it has a velocity V_{P1} relative to Mars. By definition, the probe is approaching Mars at a velocity greater than the escape velocity so it will follow a hyperbolic trajectory with Mars at the focus. The probe will leave the SOI of Mars with a velocity V_{P2}. The distance from Mars at which the probe enters the SOI will be the same as the istance at which it leaves the SOI. Basic orbital properties dictate that the magnitude of velocities V_{P1} and V_{P2} will be the same, but the direction will be deflected. We can see the geometry of the slingshot manoeuvre, referenced to Mars, in Figure 8.

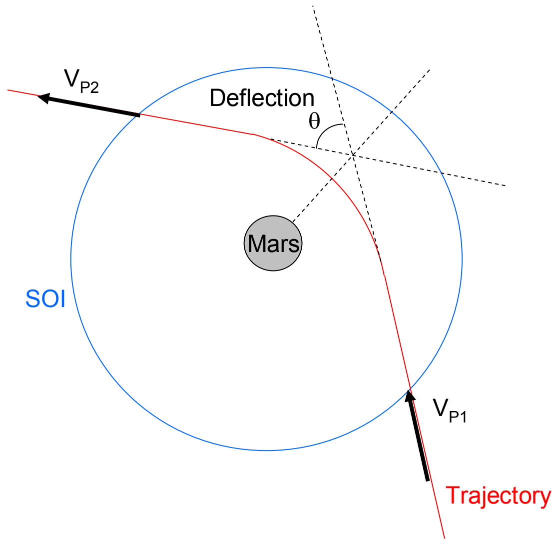


Figure 8 – Slingshot referenced to the planet

The planet Mars is, of course, moving around the sun at a velocity V_{Mars} of around 24.1 km/s. If we consider the velocity of the probe relative to the Sun, we can add the velocity vectors to see the initial and final velocities of the probe relative to the Sun. We'll denote the initial velocity V_{in} and the final velocity V_{out}.

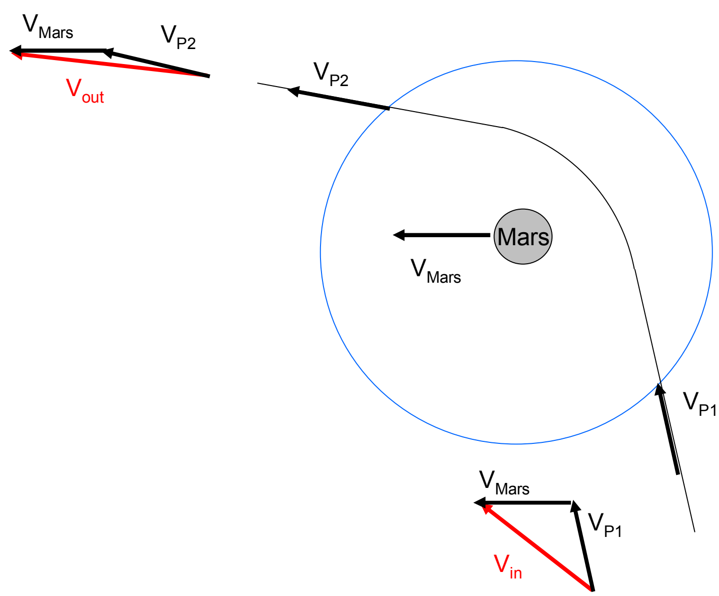


Figure 9 – Slingshot referenced to the Sun

The vector addition shows that magnitude of V_{out} is different to V_{in}. If the probe approaches Mars from “behind” as shown in Figure 9 it will depart with a higher velocity *relative to the Sun* but the same velocity relative to Mars.

Slingshots do not change the velocity relative to the planet, but only relative to a body with some velocity relative to the planet.

The slingshot is thus very useful for changing the direction of a trajectory without burning fuel, and is also useful for changing velocity relative to the Sun. This makes it ideal for manoeuvres within the solar system, and it has featured in many missions including the Voyager probes at Ulysses.

As the probe’s velocity has increased its kinetic energy must have changed relative to the Sun. Energy must be conserved, so this energy must have come from somewhere. The kinetic energy has come from the planet, deflecting its orbit by a tiny amount.