Orbits and Kepler's Laws

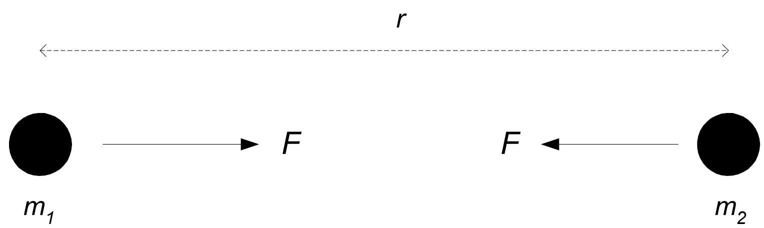
This web page introduces some of the basic ideas of orbital dynamics. It starts by describing the basic force due to gravity, then considers the nature and shape of orbits. The next section considers how velocity change (ΔV, pronounced "delta-vee") is used to initiate manoeuvres, and looks at sources of velocity change.

Having established these basic ideas it then looks at interplanetary manoeuvres, starting with a simple Earth-Moon transfer and moving on to more complex interplanetary manoeuvres. By the end of this section readers should be in awe of the guys at JPL who do this for a living!

Gravitational Force

There is much discussion among physicists about the nature of gravity. For the purpose of this website we'll use Newtonian mechanics and ignore notions of gravity waves or any other current theories.

In the Newtonian world any two lumps of matter will exert a gravitational force on each other. Imagine we have two masses *m1* and *m2*, and they are a distance *r* apart (masses in kg, distance in metres).



The attractive force between the two masses can be calculated from:

$$F = \frac{Gm_1 m_2}{r^2}$$

where G is the universal gravitational constant, G = 6.672 × 10^{-11} m^3 /kg sec^2. If one of the masses is fixed and very much greater than the other, for example a planet and spacecraft, then it is sometimes written:

$$F = \frac{GMm}{r^2}$$

where M is the mass of the plane an m is the mass of the spacecraft. The product GM is called the gravitational parameter, written as the Greek letter μ. The value of μ is different for each planet, so remember to change value when working on interplanetary manoeuvres. Some useful gravitational parameters are:

Body	μ (m^3/s^2)	μ (km^3/s^2)
Sun	1.327×10^{20}	1.327×10^{11}
Earth	3.986×10^{14}	3.986×10^{5}

Moon	4.902×10^{12}	4.902×10^{3}
Mars	4.281×10^{13}	4.281×10^{4}

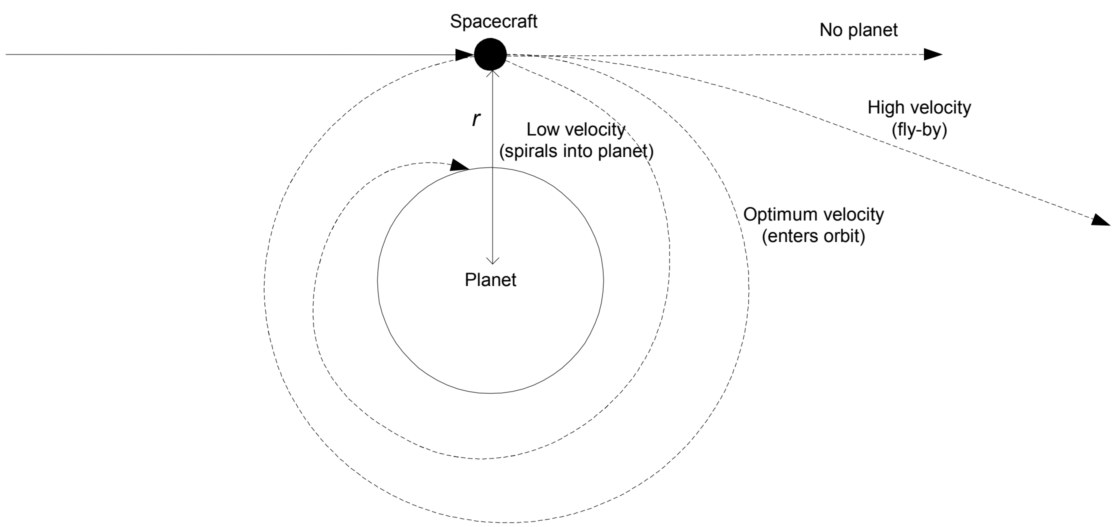
If a spacecraft was stationary above the planet it would simply fall out the sky due to gravity. In order to stay in orbit it needs to move in an orbit.

Basic orbits

Let's debunk one myth straight away – a spacecraft is not kept in orbit by centrifugal force. Let's consider Newton's first law:

"A body will remain at rest or uniform motion in a straight line unless acted on by a force"

If spacecraft is travelling in empty space with its motors off. There are no forces on the spacecraft so it will travel in a straight line, as predicted by Newton's first law. Imagine that a planet suddenly appears beneath the spacecraft. The only force introduced is gravity, and the effect of this is to deflect the path of the spacecraft towards the planet. If the spacecraft is travelling very fast its path will bend, but it will escape the gravitational force of the planet. If it is travelling very slowly it will spiral into the planet and crash. Over a narrow range of speeds it will fall towards the planet, but it will never land. It is in orbit.



One way of thinking about this is that the curvature of the planet is such that it falls away at the same rate as gravity is pulling the spacecraft towards it. The spacecraft is in "free fall" around the planet.

If the spacecraft is travelling with a velocity v m/s at a distance of r metres from the centre of the planet, then a circular orbit will occur only when the following equation is true:

$$v = \sqrt{\frac{\mu}{r}}$$

If the spacecraft is travelling slightly faster than this it will just fail to escape the gravitational pull and remain in an orbit which is not circular but elliptical. It can be shown that, if the velocity is equal to:

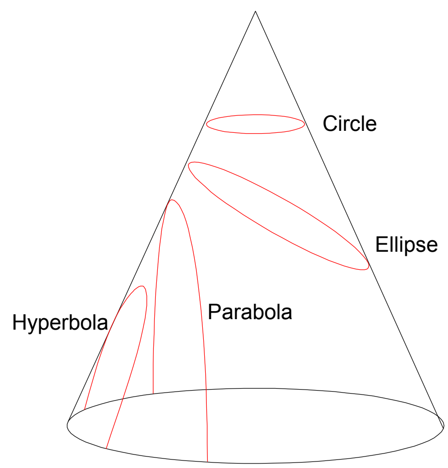
$$v = \sqrt{\frac{2\mu}{r}}$$

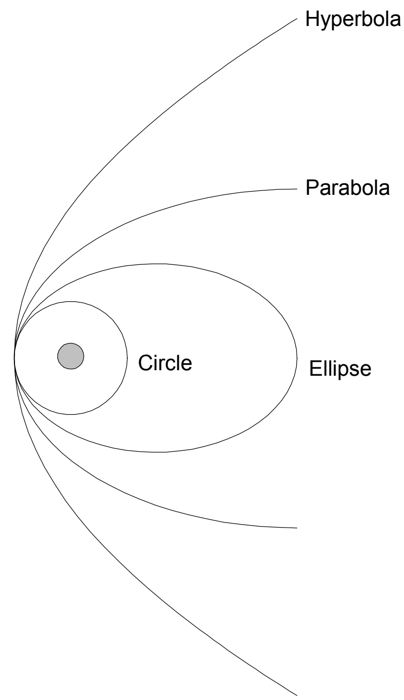
then it will just escape from the planet's gravitational pull. The shape of the orbit will be parabolic. If we exceed this velocity the shape of the orbit becomes hyperbolic and the spacecraft escapes from the planet's gravitational pull at a higher velocity.

We can summarise the shape of orbits in the following table:

Spacecraft Velocity	**Orbital shape**	**Comments**
$v < \sqrt{\frac{\mu}{r}}$	Unsustainable	Spacecraft spirals into planet
$v = \sqrt{\frac{\mu}{r}}$	Circular	Closed orbit
$\sqrt{\frac{\mu}{r}} < v \leq \sqrt{\frac{2\mu}{r}}$	Elliptical	Closed orbit
$v = \sqrt{\frac{2\mu}{r}}$	Parabolic	Open orbit, minimum escape velocity
$v > \sqrt{\frac{2\mu}{r}}$	Hyperbolic	Open orbit, escape velocity

The shapes of orbits are either circles, ellipses, parabolae or hyperbolae. These shapes may appear to be very different but mathematically they all have something in common: they are derived from the "cuts" of a cone. The orbital shapes are thus referred to as "conic sections".





Later on we'll see that these "conic section" orbit shapes have a mathematical relationship to each other, but for now we'll just accept this as fact.

Keplers laws

Between 1609 and 1619 **Johannes Kepler** published his famous 3 laws of planetary motion, based on observations made by the astronomer **Tycho Brahe**. Kepler's laws described the orbits of bodies to remarkable accuracy, and stated that all orbits would be elliptical or circular. In 1687 **Sir Isaac Newton** supplied the theoretical explanation for why the orbits were this shape and allowed calculation of the velocities which a satellite would need to reach if it was to sustain an orbit.

Kepler's laws state:

1. A body orbiting around a planet will describe an orbit that is an ellipse with the planet at one of the foci.

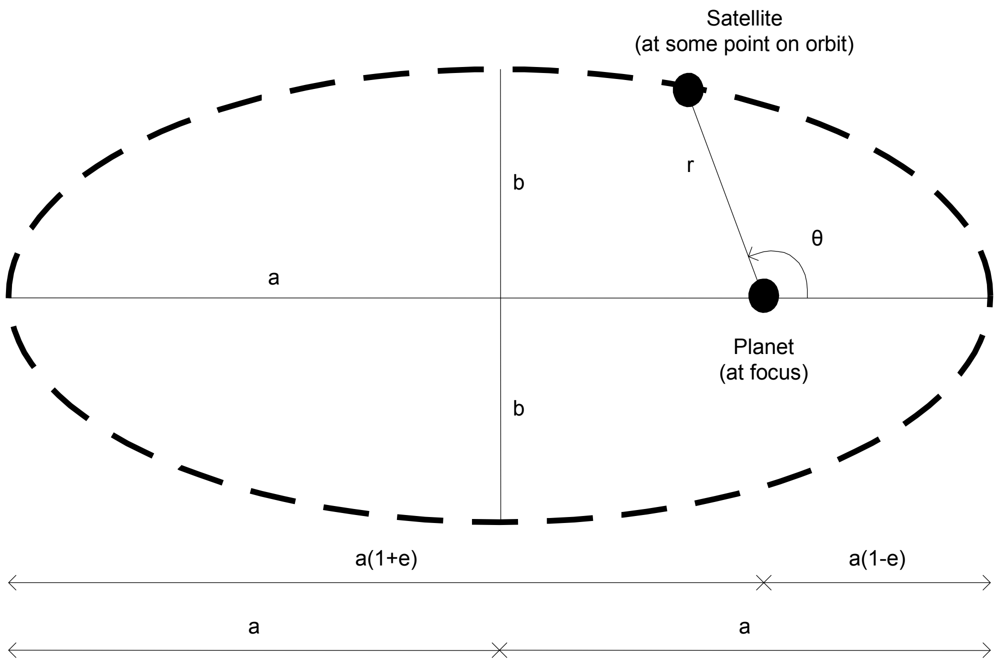
2. If we draw a line from the planet to the body in orbit around it, the line will sweeps out equal areas in equal intervals of time.

3. The square of the time taken for a body to complete one orbit is proportional to the cube of the major axis of the orbit.

What do these mean in practice?

Kepler's First Law

Kepler's first law describes the shape of an orbit, based on observations of the motions of planets.



The geometry of an ellipse tells us that it is an "eccentric" circle, where the degree of eccentricity is denoted by the letter *e*. The value of e is defined from the dimensions of the ellipse *a* and *b*, by the equation:

$b^2 = a^2 (1-e^2)$

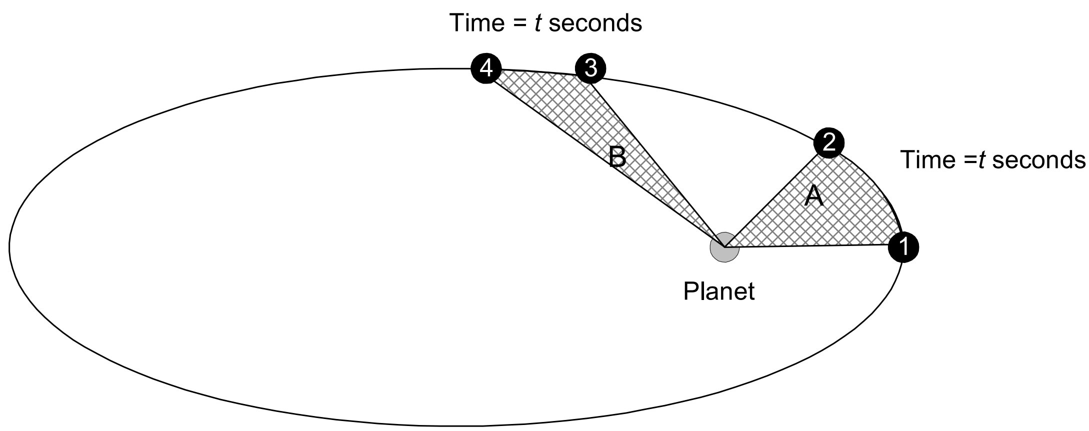
From the geometry of an orbit it can be shown that, for a satellite at any point on the orbit at a distance *r* from the planet, the following relationship is always true:

$$r = \frac{a(1-e^2)}{1+e\cos(\theta)}$$

Let's think about these equations a bit. If the eccentricity is zero, then what do we get? Substituting e=0 into the first equation we find that a=b. Form the second equation we find that r=a. Both of these cases indicate that we have a circle when e=0, so a circular orbit is just a special case of an elliptical orbit where the eccentricity is zero.

Kepler's Second Law

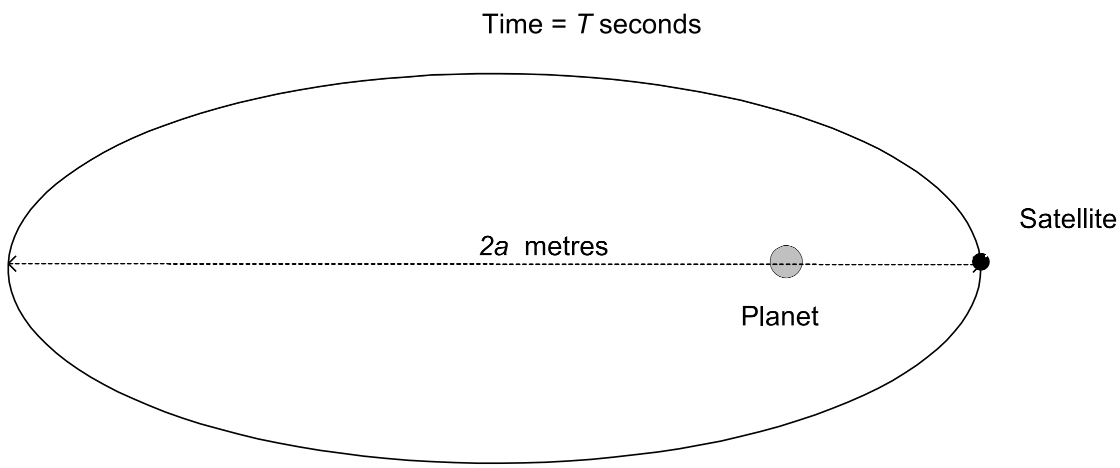
Imagine our satellite is at point ❶ in its orbit around a planet. A time *t* seconds later it has moved on to point❷. The line between the planet and the satellite will sweep out an area A in that time. Later on in its orbit it moves from point ❸ to point ❹ in a time *t*, sweeping out an area B.



Kepler's second law tells us that area A will always equal area B, regardless of where we start to measure the time *t*.

Kepler's Third Law

The diagram below shows an elliptical orbit with a major axis *a*. It takes a time, which we'll call *T* seconds, to complete one orbit of the planet. Instinct says that if we make the orbit bigger, in other words increase *a*, the satellite will take longer to complete one orbit.



Kepler's third law tells us how *T* varies as we change *a*:

$$T = 2\pi\sqrt{\frac{a^3}{\mu}}$$