# Orbital Manoeuvres

All manoeuvres depend changes in velocity (accelerations) which, from Newton's first law, require the action of a force. Generally, manoeuvres are the result of thrust from a spacecraft's motors. In this section we'll consider simple manoeuvers in the same orbital plane, then consider changes of plane.

## *Orbital Manouevres in the Same Plane*

By considering the kinetic and potential energy of a spacecraft, it is possible to relate its velocity and position to the constant $\mu$. If the spacecraft is travelling with a velocity $v$ along the direction of the orbit, it can be shown that:

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

This equation can be used to calculate the velocities required for different orbits, and hence the change in velocity $\Delta V$ required to change between those orbits. If, for example, a satellite is in an orbit with semi major axis $a$=8000 km, and the satellite is 12000 km from the Earth, it's velocity must be:

$$v^2 = 3.986\times10^5\left(\frac{2}{12000} - \frac{1}{8000}\right)$$

<u>$v$ = 4.1 km/s</u>

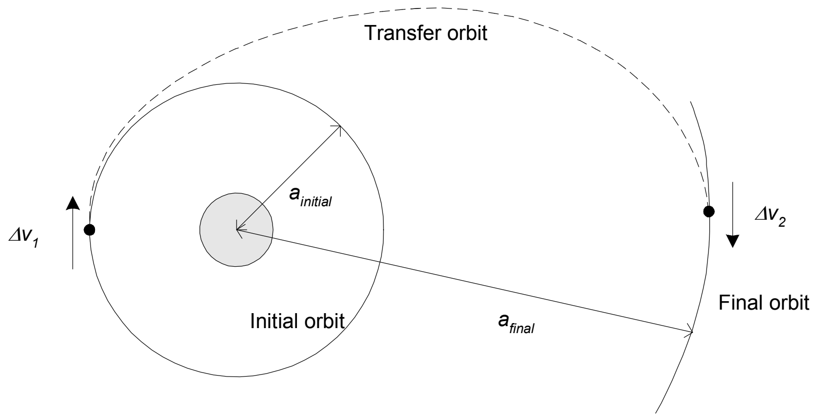
In this simple example we've just considered the velocity given that the orbit was known ($a$=8000 km). Like all good equations, we can calculate any variable from knowledge of the other two. If we know the position of the satellite (distance $r$) and its velocity $v$ we can establish the semi major axis of the orbit $a$.

A classical manoeuvre problem would be to have an orbit with a known semi major axis $a_1$, then change the velocity of the spacecraft to change this to a new value $a_2$. The short motor burn which causes the change in velocity takes place with a value of r which is constant to both orbits. We thus know $a_1$, $a_2$ and $r$, so we can calculate the velocity required for both orbits, and hence the change in velocity to move between the two orbits.

A simple example of this is the move between two circular orbits, commonly called the Hoffman transfer.

## *Hoffman Transfer*

The Hoffman Transfer is a two-stage transfer between two circular orbits. the first "burn" $\Delta V_1$ changes the velocity so that the orbit becomes an ellipse which intersects both circular orbits. The second "burn" $\Delta V_2$ changes the velocity so that the spacecraft enters the new circular orbit.




Lets consider a Hoffman transfer from a low orbit ($a_1$ = 8000 km) to geostationary orbit ($a_2$ = 42000 km).

To calculate the velocity before burn 1 we simple calculate the velocity for a circular orbit ($r = a_1$).

$$v^2 = \mu\left(\frac{2}{a_1} - \frac{1}{a_1}\right) \Rightarrow v = \sqrt{\frac{\mu}{a_1}} = \sqrt{\frac{3.986 \times 10^5}{8000}} = 7.06 \text{ km/s}$$

The velocity after burn 1 we know that $r = a_1$ and the semi major axis of the elliptical transfer orbit must be:

$$a_e = \frac{1}{2}(a_1 + a_2) = ½\ (8000 + 42000) = 25000 \text{ km}$$

Thus:

$$v^2 = \mu\left(\frac{2}{a_1} - \frac{1}{a_e}\right) \Rightarrow v = \sqrt{\mu\left(\frac{2}{a_1} - \frac{1}{a_e}\right)} = \sqrt{3.986 \times 10^5\left(\frac{2}{8000} - \frac{1}{25000}\right)} = 9.15 \text{km/s}$$

Thus $\Delta V_1 = 9.15 - 7.06 =$ <u>2.09 km/s</u>

So the first burn, which puts the spacecraft into the transfer orbit, needs to accelerate the spacecraft by 2.09 km/s. Note that this is a positive acceleration so the velocity increases. If the answer had been negative then the velocity would have to decrease. The $\Delta V$ is always an increase to go to higher orbits and a decrease to go to, lower orbits.

We can do a similar exercise for the second burn. Immediately before the burn the spacecraft's velocity can be calculated since $r = a_2$ and $a = a_e$. hence:

$$v^2 = \mu\left(\frac{2}{a_2} - \frac{1}{a_e}\right) \Rightarrow v = \sqrt{\mu\left(\frac{2}{a_2} - \frac{1}{a_e}\right)} = \sqrt{3.986 \times 10^5\left(\frac{2}{42000} - \frac{1}{25000}\right)} = 1.74 \text{ km/s}$$

On reaching the apogee of the transfer orbit the spacecraft has been decelerated under earth's gravity. To put the spacecraft into its final orbit , where $r = a_2$ and $a = a_2$, it needs to be travelling at a velocity:

$$v^2 = \mu\left(\frac{2}{a_2} - \frac{1}{a_2}\right) \Rightarrow v = \sqrt{\frac{\mu}{a_2}} = \sqrt{\frac{3.986\times10^5}{42000}} = 3.08 \text{ km/s}$$

Thus the change in velocity is

$\Delta V_2 = 3.08 - 1.74 =$ 1.34 km/s

Note that this is also a positive velocity change.

With these two "burns" we have raised the spacecraft from 8000 km to 42000 km. The total ΔV was 3.43 km/s. As all ΔV is achieved by burning fuel, the important question is could we have raised the spacecraft using less fuel? The answer is no. The Hoffman transfer is the most fuel efficient transfer between orbits.

## *Escape Manoeuvres*

Imagine or spacecraft was in its orbit at 8000 km and we wanted to achieve escape velocity. This is relatively straightforward as escape velocity simply means that $r = 8000$ and $a = \infty$. Substituting this into our equation:

$$v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \Rightarrow v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)} = \sqrt{3.986\times10^5\left(\frac{2}{8000} - \frac{1}{\infty}\right)}$$

since $\frac{1}{\infty} = 0$

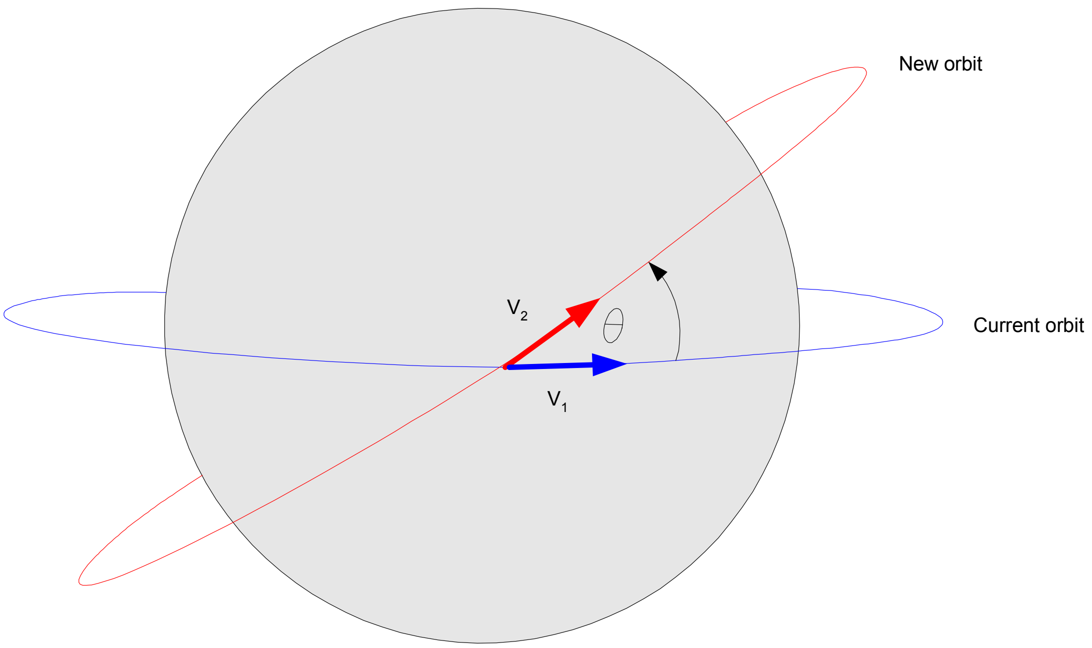
$$v = \sqrt{3.986\times10^5\left(\frac{2}{8000} - 0\right)} = 9.98 \text{ km/s}$$

So the ΔV required to reach escape velocity is 9.98 – 7.06 = 2.92 km/s

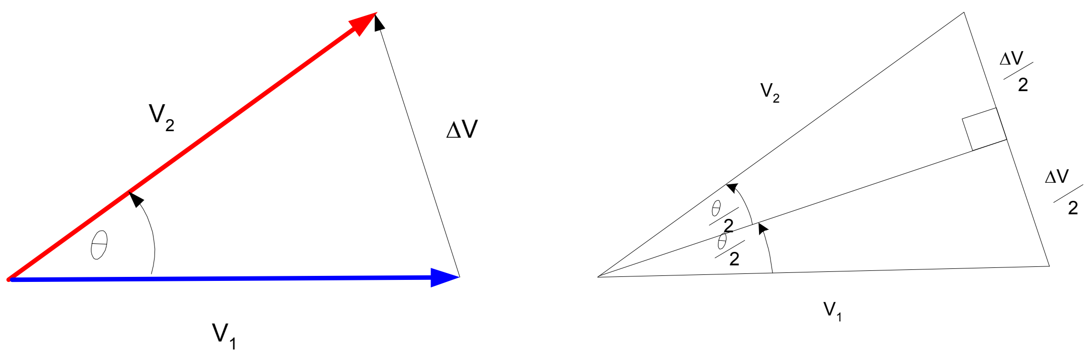
## *Change of Orbital Plane*

The concept of ΔV applies to changes in direction as well as speed. Imagine we have a satellite in orbit around the Earth, and we want to put it into a new orbital plane, but at the same altitude. How much of a velocity change do wee need to give it, in what direction, and when?

The following diagram answers the question "when". We must carry out the "burn" to change velocity when the old orbit (in blue) crosses the new orbit (in red).




As the satellite is staying at the same altitude we know that the magnitude of the velocity vectors $v_1$ and $v_2$ will be the same, but the direction will be different. To work out the velocity vector ΔV to effect a change the direction through an angle θ we simply construct a vector triangle.




Applying simple trigonometry to the velocity we can show that the magnitude of the velocity vector ΔV is:

$$\Delta v = 2V \sin\frac{\theta}{2}$$

The direction of the burn is in a direction of 90+θ/2 degrees from the direction of travel of the satellite.

What does this meen in reality? Imagine our satellite has a mass of 1000kg, which is quite small for a modern communications satellite, and is travelling at 8km/s. To change its direction by 5 degrees we need to give it a $\Delta V$ of magnitude:

$$\Delta v = 2V \sin\frac{\theta}{2} = 2 \times 8000 \times \sin\frac{5}{2} \approx 700 m/s$$

If we revisit the section on the rocket equation, and assume that our booster motor on the satellite has a specific impulse of 250 seconds, we can calculate how much fuel we need to burn to execute this orbit change:

$$\Delta v = v_e \ln\left(\frac{m_{sat} + m_{fuel}}{m_{sat}}\right)$$

It can be shown that the rocket exhaust velocity, $v_e$, can be found from the equation:

$$v_e = g_o I_{sp}$$

Where $g_o$ is the acceleration due to gravity at sea level. Thus:

$$\Delta v = I_{sp} g_o \ln\left(\frac{m_{sat} + m_{fuel}}{m_{sat}}\right) \Rightarrow 700 = 250 \times 9.81 \times \ln\left(\frac{1000 + m_{fuel}}{1000}\right)$$

From which we can determine that $m_{fuel} =$ 330 kg. To put this into perspective, we have to burn about a third of the mass of the satellite as fuel to execute a 5 degree change in direction. It can be seen that orbit change manoeuvres are very fuel intensive, and are thus carried out very infrequently.