

Physics of Rocket Flight

In order to understand the behaviour of rockets it is necessary to have a basic grounding in physics, in particular some of the principles of statics and dynamics. This section looks at the relationships between distance, velocity, acceleration, force, work, impulse, energy and power. These basic ideas are often encountered in rocket science, and it is useful to have a knowledge of how they interrelate.

Distance, Velocity & Acceleration

We all understand the concept of distance. We can draw two points and measure the distance between them. If we know the time it takes to travel from one point to the next, and we know the distance, we can calculate our speed. Or can we?

What we haven't taken into account is the path we follow. For example, if we travel from London to Bath, about 100 miles, in 5 hours we would say that we travelled at an average speed of 20 mph. This, however, is incomplete information. If the journey took us via Birmingham it may be that we travelled 250 miles, so our average speed was 50 mph. Clearly we need to think a little bit more about what we mean by distance and speed.

In physics it is normal to call the distance between two points in a straight line the *displacement*. The term "displacement" not only considers the distance between the points but also the direction, thus it is a vector quantity. Speed also has an associated direction in physics, and is thus a vector quantity called *velocity*. A change in either the magnitude or the direction of the velocity vector is called *acceleration*. Most people can identify with a change in the magnitude of the velocity vector due to acceleration, but why the direction?

Newton's first law tells us that "a body will remain at rest or uniform motion in a straight line unless acted on by a force". If we have a force acting on the body it could change its motion from "uniform motion in a straight line" by either changing the uniform motion (magnitude), moving it off the straight line (direction) or both. At any instant of time a body which is being accelerated has an *instantaneous velocity*.

Imagine a model rocket. In terms of Newton's first law it is "at rest" immediately before launch. On ignition of the motor it is acted on by a force (the thrust of the motor) and accelerates upwards. During its flight it may be acted on by other forces as it pushes air molecules aside (drag) or is pressed by crosswinds. These result in accelerations which may deflect it from its path, or change the magnitude of its velocity.

Sometimes we use calculus to express displacement, velocity and acceleration. Calculus was another of Newton's contributions to science. If we consider travel over a distance r ,

we can define velocity v as the rate of change of distance, and acceleration a as the rate of change of velocity. In mathematical terms:

Distance: r

Velocity: $v = \frac{dr}{dt}$

Acceleration: $a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$

Distance is always measured in meters (m), velocity is measured in meters/second (m/s) and acceleration in meters/second/second (m/s^2). Having grasped these basic concepts of displacement, velocity, and acceleration, let's go on and consider forces.

Equations of Motion

The equations of motion for a body under a constant acceleration are often encountered in physics. If we consider a body travelling at a velocity v_1 m/s for a time t sec, we know that it will travel a distance of r meters. We calculate this using the equation:

$$r = v_1 t$$

If we apply a force to the body we will cause the body to accelerate. We can add this factor into the equation:

$$r = v_1 t + \frac{1}{2} a t^2$$

If we differentiate this equation with respect to time we get:

$$\frac{dr}{dt} = \frac{d(v_1 t)}{dt} + \frac{1}{2} \frac{d(at^2)}{dt}$$

which simplifies to:

$$v_2 = v_1 + at$$

So if we know the acceleration and initial velocity v_1 we can find the velocity v_2 at any subsequent time. By rearranging the above equations we get the final equation:

$$v_2^2 = v_1^2 + 2ar$$

These equations are very useful, but also very misleading. They only apply to motion under a constant acceleration, whereas in rocketry the acceleration is seldom constant. The equations become useful when using numerical methods to approximate the motion of a body under varying accelerations. In these circumstances the acceleration is viewed as “instantaneously constant” over a very short interval of time, typically thousandths of a second.

They should NOT be used in general rocket analysis as the results can be highly misleading.

Force

Newton’s second law defines force as:

Force = mass x acceleration

In its convenient mathematical form:

$$F = ma = m \frac{dv}{dt} \dots\dots\dots \text{equation 1}$$

If we apply a force to a body with low mass (light), the one with high mass (heavy) we’ll find that the light body will accelerate more. Another way of thinking about force is to think about acceleration. If we want the two bodies to accelerate at the same rate we must apply a larger force to the heavier body. Force has other more subtle definitions. One used in rocketry is:

Force = rate of change of momentum

This is a variation of equation 1 in which neither mass nor acceleration are constant. This is the case in rockets where the mass decreases as fuel is burned. Mathematically we can express this as:

$$F = \frac{d(mv)}{dt}$$

The correct unit for measuring force is the kilogram-meter/second/second. This is a bit of a mouthful so, as is common in science the unit was named after a great scientist. The unit of force is thus the “Newton” (N).

Weight is also a force, however there is a common misconception the unit of weight is the kilogram. In fact this is the unit of mass, to convert this to weight (a force) the mass must be subjected to some acceleration. On Earth, this acceleration is due to gravity and has a value of 9.81 m/s². A rocket with a mass of 2 kg thus has a weight of 19.62 N.

This is important when considering the rating of motors where the “weight” of the rocket on the scales is in fact its mass. The weight is about 10 times more.

Example: to safely launch a rocket a thrust to weight ratio of at least 5:1 is required. The rocket weighs 0.4 kg, and the motor has a thrust of 15N. Is this safe?

Answer: The weight of the rocket is $0.4 \times 9.8 = 3.9$ N. Thrust is 15 N. The thrust to weight ratio is thus 15:3.9, or 3.8:1. This is less than 5:1 so the launch would be unsafe.

Momentum

From the previous section we can see that momentum can be defined as:

Momentum = mass x velocity

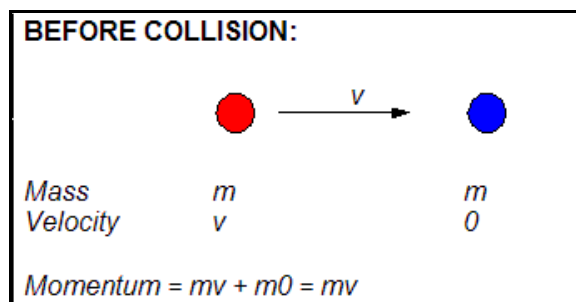
The momentum of an object is simply the product of its mass and velocity. A rocket with a mass of 2 kg travelling at 100m/s has a momentum of 200 kgm/s. Similarly a car with a mass of 1000kg travelling at 30 m/s has a momentum of 30,000 kgm/s. We often use P to represent momentum, thus:

$$P = mv$$

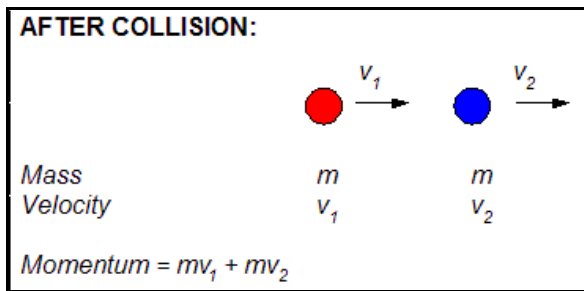
From our definition of force above, we can see that:

$$F = \frac{d(mv)}{dt} = \frac{dP}{dt}$$

Momentum has an interesting property in that it is always conserved. It can thus be transferred from one object to another. Imagine a red snooker ball of mass m travelling along a table with a velocity v , where it hits a stationary blue ball of mass m . Before the impact the total momentum is mv plus the momentum of the second ball (zero, since its velocity is zero).



After the impact both balls are moving. There is an important principle of physics which says that the total momentum in a system is constant, in other words it is conserved. If we look at a diagram of the balls after the collision:



The red ball and the blue ball are travelling with velocities v_1 and v_2 respectively. The momentum of the red ball is thus mv_1 and that of the blue ball is mv_2 . As momentum is conserved, the momentum in the “system” before the collision is equal to the momentum in the system after the collision, thus:

$$mv = mv_1 + mv_2$$

The derivation of the rocket equation makes use of the property of conservation of momentum.

Energy

Energy, like momentum, is conserved. It cannot be created or destroyed (although Einstein demonstrated that it can be converted into mass). If we look at a “system” which has no energy entering or leaving it (called a “closed system” in physics), then the total energy in the system will be constant. The energy within the system may change form, but the total energy will be constant.

The unit of energy is the Joule.

In rocketry we are concerned with three main types of energy: kinetic energy (KE), potential energy (PE) and chemical energy. Kinetic energy is the energy of a moving object, and can be easily calculated if we know the mass m of the object and its velocity v :

$$KE = \frac{1}{2}mv^2$$

A rocket with a mass of 10kg travelling at a velocity of 100 m/s has a KE of $\frac{1}{2} \times 10 \times 100^2$ Joules, or 500,000 Joules.

Potential energy describes the energy which a rocket has by virtue of its position. A rocket 10 metres off the ground has energy which could be released if it were allowed to fall. If the rocket was dropped, the PE would fall to zero when it landed. PE can be

calculated easily: If an rocket of mass m is held at a height h above the ground (we'll assume that PE is zero at ground level), then the PE can be calculated from:

$$PE = mgh$$

The value of g is the acceleration due to gravity (9.81 m/s^2). We know that the mg term is a force and is thus measured in Newtons, since it is mass multiplied by acceleration. The height h is measured in meters. We can thus see that potential energy is measured in Newton-meters, and that these are the same as Joules.

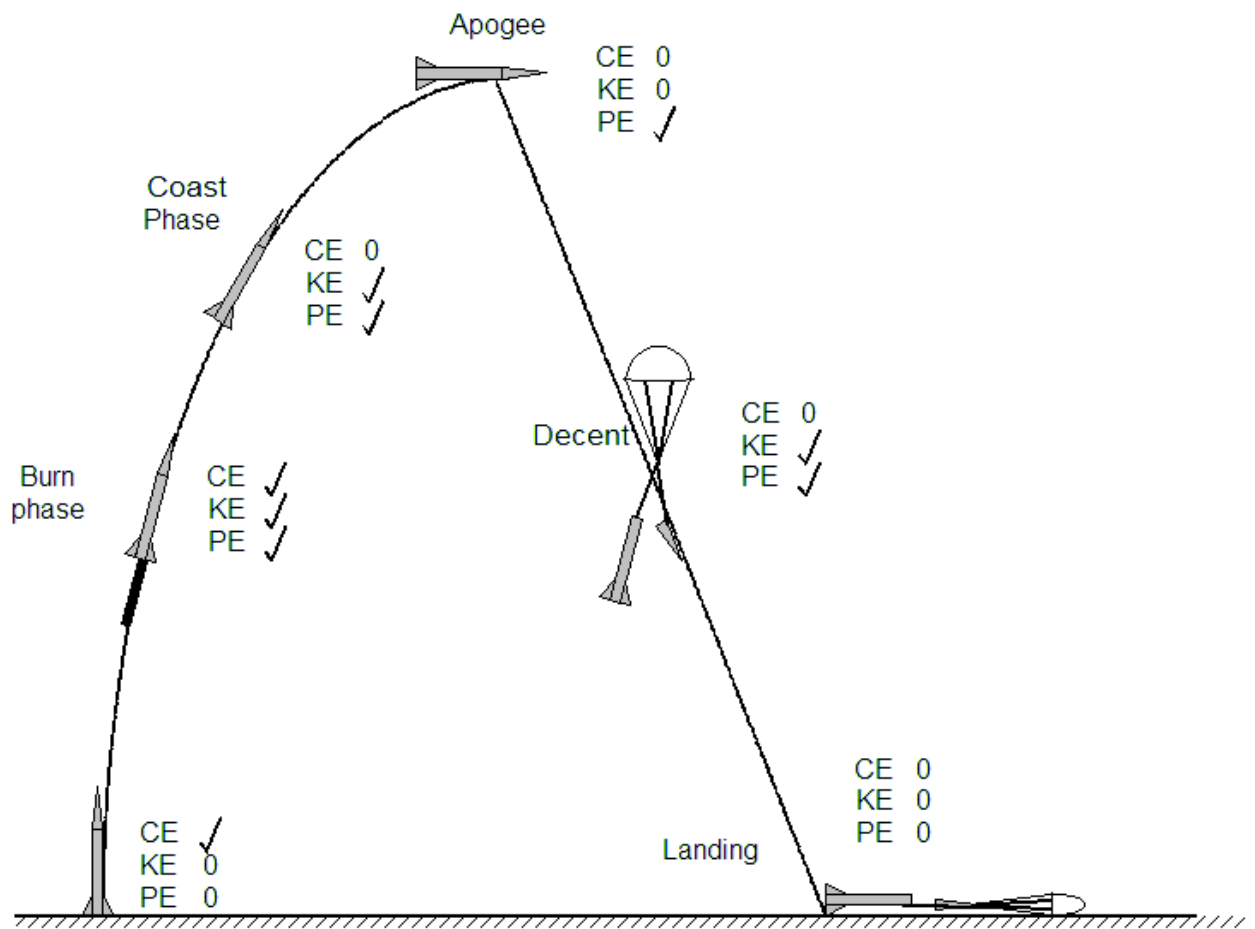
Another form of energy, chemical energy, is more complex. It is the energy "stored" in the chemical bonds within the propellant, and released when the propellant is burned. The chemical energy released for every molecule liberated by the chemical reactions in the propellant can be approximately calculated as:

$$CE = \frac{5}{2}kT$$

The term k is Boltzmann's Constant ($1.38 \times 10^{-23} \text{ J/K}$) and T is the absolute temperature of the chemical reaction. It can be seen that the hotter the reaction, the more chemical energy is released.

Putting all this together, we can start to consider the energy in a rocket flight. When the rocket is on the launch pad it is at height of 0 m, and is travelling at 0 m/s. It thus has no PE or KE, but it does have a lot of CE stored in the propellant.

After ignition, and during the burn phase, the rocket loses some of this CE, but acquires PE and KE as it gains height and speed. At burnout the CE goes to zero as there is no propellant left to burn, and the KE is at a maximum as the rocket is travelling at its maximum velocity. When the rocket reaches apogee it has no velocity for an instant, it thus has no KE, no CE, but has the maximum amount of PE. In descent, at a constant rate, the rocket has a constant KE and diminishing PE. Finally at landing the rocket has no PE, no KE and no CE.



Hang on, didn't we start by saying that energy was conserved? We've just seen a rocket start with a load of CE and finish with nothing! So where did all the energy go?

The answer is that the rocket is not a "closed" system. Energy has left the rocket in many forms:

- As heat, passed to the atmosphere by the hot exhaust gases
- As sound, the energy was passed to the atmosphere as pressure which we detected as sound
- As turbulence, as the rocket and its parachute moved the air

If we flew the rocket in a large, closed, room and measured all the different energy outputs we would find that the total energy in the room would remain constant. Energy would be conserved.

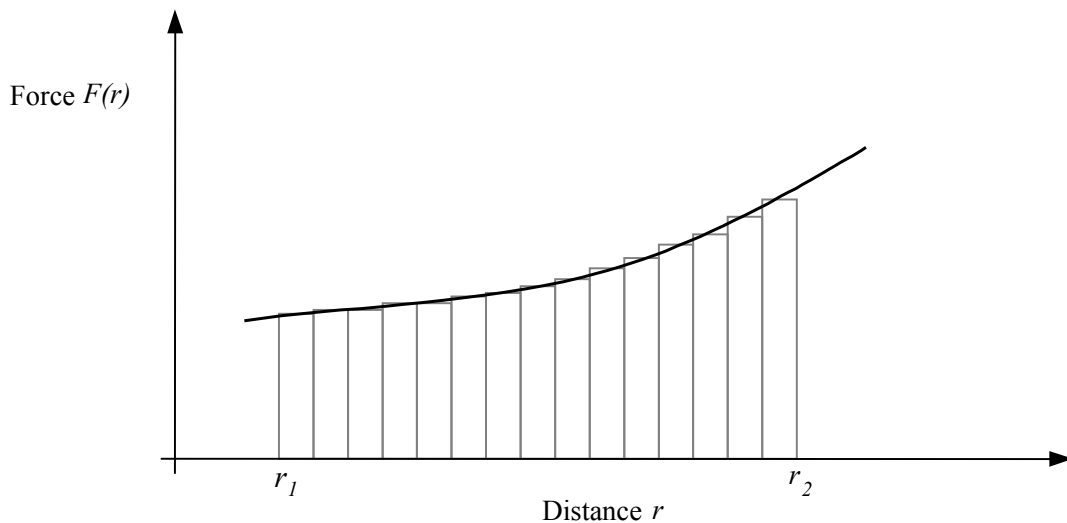
Work

The concept of "work" gets relatively little attention in physics, and yet it provides an essential link between force and energy. It also allows us to shortcut the analysis of

systems where the forces, and hence accelerations, are not constant. At its simplest level, and assuming a constant force, we can define work by considering a force F acting on a body and causing the body to be displaced by a distance r . The work done of the body is the product of the displacement and the force in the direction of the displacement.

$$W = Fr$$

To understand the usefulness of work we need to consider more complex systems. The forces acting on a rocket will vary with time. We can consider a force which varies with position.



We denote the force at a position r as $F(r)$, so the work done at position r is:

$$W(r) = F(r)\Delta r$$

The work done by the force over the distance r_1 to r_2 can be approximated by adding up all the work done over each element Δr . As we make Δr progressively smaller, this addition becomes integration, thus we can calculate the work as:

$$W = \int_{r_1}^{r_2} F(r)dr$$

Work is measured in Newton-meters. We know from considering potential energy that it is also measured in Newton-meters, and that this energy is measured in Joules. Are work and energy similar? The answer is yes. We can readily show a relationship between work and another form of energy, kinetic energy.

We know that:

$$F = m \frac{dv}{dt}$$

Thus we can say that:

$$W = \int m \frac{dv}{dt} dr = \int m \frac{dr}{dt} dv$$

We saw earlier that velocity is the rate of change of distance, so that:

$$W = \int m \frac{dr}{dt} dv = \int mv \cdot dv = \frac{1}{2} mv^2$$

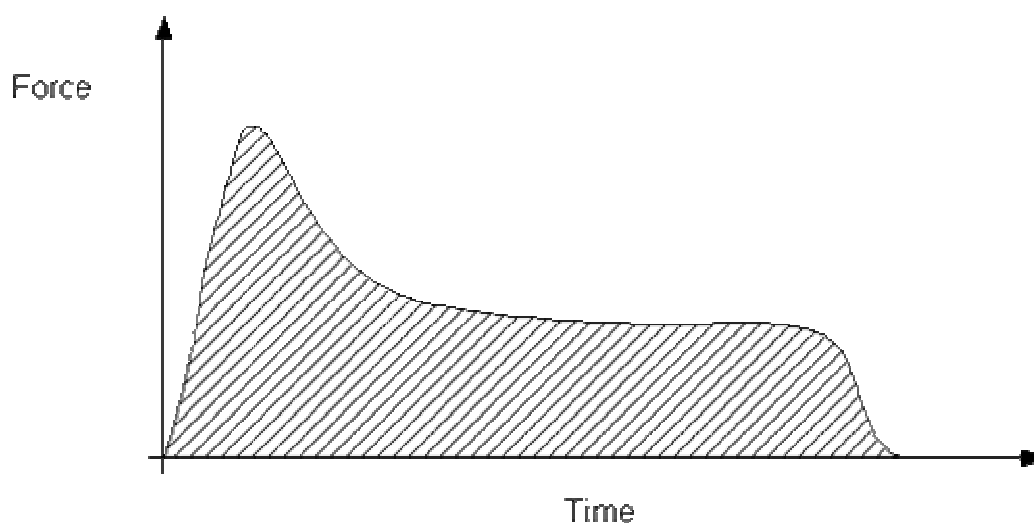
Work and energy are thus very closely related. When we “do work” due to a force in the direction of travel we give the body energy. Conversely, forces opposing the direction of travel rob the body of energy. Forces at right angles to the direction of travel do no work.

Impulse

Impulse is the product of Force and time. If we apply a force F to a body for a time t the impulse is:

$$I = Ft$$

Of course this assumes a constant force. In rocketry the net force varies with time as the mass and acceleration of the rocket are not constant. In these circumstances the total impulse can be regarded as the area under the force/time curve:



The impulse is the shaded area. Mathematically,

$$I = \int F dt$$

Sometimes in rocketry we use the term “specific impulse”. This is used to describe the impulse provided by 1 kg of propellant. This term is used more in British text books than American ones.

Power

Power is defined as:

Power = the rate of use of energy = the rate at which work is done

Power is measured in Joules/second, also known as Watts. Another unit named after a great scientist. A common misconception is that a motor with “G” impulse is twice as powerful as one with “F” impulse. In fact it has twice as much impulse, not power. If we imagine a motor which gives a thrust of F Newtons for t seconds, it has an impulse of:

$$I = Ft \quad \text{Newton-seconds}$$

If it moves the rocket r meters in that time, it does W Joules of work where:

$$W = Fr \quad \text{Joules}$$

The power of the rocket P is the energy divided by the time, in other words the rate of doing work, so that:

$$P = \frac{W}{t} = \frac{Fr}{t} \quad \text{Watts}$$

From which we can derive a relationship between power and impulse:

$$P = I \frac{r}{t^2}$$

In other words, the power of a rocket depends on the impulse, the distance over which the motor burns, and the square of the time of the burn. Impulse and power are clearly NOT the same thing!