**Rocket Propulsion**

In the section about the rocket equation we explored some of the issues surrounding the performance of a whole rocket. What we didn’t explore was the heart of the rocket, the motor. In this section we’ll look at the design of motors, the factors which affect the performance of motors, and some of the practical limitations of motor design. The first part of this section is necessarily descriptive as the chemistry, thermodynamics and maths associated with motor design are beyond the target audience of this website.

**General Principles of a Rocket Motor**

In a rocket motor a chemical reaction is used to generate hot gas in a confined space called the combustion chamber. The chamber has a single exit through a constriction called the throat. The pressure of the hot gas is higher than the surrounding atmosphere, thus the gas flows out through the constriction and is accelerated.

![Combustion chamber, Throat, Nozzle](image)

It sounds simple, so why is rocket science so complex? Well, firstly there’s chemistry and the selection of the right reagents from many thousands of possibilities. Then there’s the design of the motor to make it capable of withstanding the temperatures and pressures of the reaction while still being as light as possible. There’s also the design of the throat and nozzle to ensure that the exhaust velocity is as fast as possible. Putting all these bits together, the average rocket scientist needs (as a minimum) to understand chemistry, mechanical engineering, thermodynamics, materials science and aerodynamics.

**Propellants**

The chemical reaction in model rocket motors is referred to as an “exothermal redox” reaction. The term “exothermal” means that the reaction gives off heat, and in the case of rocket motors this heat is mainly absorbed by the propellants raising their temperature. The term “redox” means that it is an oxidation/reduction reaction, in other words one of the chemicals transfers oxygen atoms to another during the reaction (OK chemists, I know that this is not a comprehensive definition but it will suffice!). The two chemicals are called the oxidising agent and the reducing agent.

The most popular rocket motors are black powder motors, where the oxidising agent is saltpetre and the reducing agents are sulphur and carbon. Other motors include Potassium or ammonium perchlorate as the oxidising agent and mixtures of hydrocarbons.
and fine powdered metals as the reducing agents. Other chemicals are often added such as retardants to slow down the rate of burn, binding agents to hold the fuel together (often these are the hydrocarbons used in the reaction), or chemicals to colour the flame or smoke for effects. In hybrid motors a gaseous oxidiser, nitrous oxide, reacts with a hydrocarbon, such as a plastic, to produce the hot gas.

**Energy Conversion**

This reaction releases energy in the form of heat, and by confining the gas within the combustion chamber we give it energy due to its pressure. We refer to the energy of this hot pressurised gas as its “enthalpy”. By releasing the gas through the throat the rocket motor turns the enthalpy of the gas into a flow of the gas with kinetic energy. It is this release of energy which powers the rocket. So the energy undergoes two conversions:

- Chemical energy to enthalpy
- Enthalpy to kinetic energy

The conversion from chemical energy to enthalpy takes place in the combustion chamber. To obtain the maximum enthalpy it is clearly important to have a reaction which releases lots of heat and generates lots of high energy molecules of gas to maximise pressure. There is clearly a limit to the temperature & pressure, as the combustion chamber may melt or split if these are too high. The designer has a limitation placed on his choice of reagents in that the reaction must not heat the combustion chamber to a point where it is damaged, nor must the pressure exceed that which the chamber can survive.

Changing enthalpy to kinetic energy takes place in the throat and the nozzle. Our mass of hot gas flows into the throat, accelerating as the throat converges. If we reduce the diameter of the throat enough, the flow will accelerate to the speed of sound, at which point something unexpected occurs. As the flow diverges into the nozzle it continues to accelerate beyond the speed of sound, the increase in velocity depending on the increase in area. This type of nozzle is called a De Laval nozzle.

![Diagram of Throat and Nozzle](image)

You will recall that the kinetic energy of a body can be calculated from:

$$KE = \frac{1}{2}mv^2$$
If we consider a small volume of gas, it will have a very low mass. As we accelerate this gas it gains kinetic energy proportional to the square of the velocity, so if we double the velocity we get four times the kinetic energy. The velocity of the supersonic flow increases proportional to the increase in area of the nozzle, thus the kinetic energy increases by the fourth power of the increase in nozzle diameter. Thus doubling the nozzle diameter increases the kinetic energy by 16 times! The De Laval nozzle make rocket motors possible, as only such high velocity flows can generate the energy required to accelerate a rocket.

In model rockets the reaction is chemical generally short lived, a few seconds at most, so the amount of heat transferred to the structural parts of the motor is limited. Also, the liner of the motor casing acts to insulate the casing from the rapid rise in temperature which would result from a reaction in direct contact with the metal casing. Model rocket motors also run at quite low pressure, well below the limits if the motor casing, further protecting the casing. It can be seen that the enthalpy of a model rocket motor is thus quite low. In large launch vehicles such as Ariane, the pressure and temperature are high, the burn may last several minutes, and the mass budget for the designer is very tight. Designing motors for these purposes is highly complex.

**Thrust**

The basic principles of a rocket motor are relatively straightforward to understand. In rocketry the motor exists to accelerate the rocket, and thus it has to develop a force called “thrust”. One of several definitions of force is that:

\[ \text{Force} = \text{rate of change of momentum} \]

If we ignore (for a few paragraphs) any external effects we can say that the thrust is entirely due to the momentum of the propellant, a force called the “momentum thrust”. If we denote the thrust as \( F \) and the momentum as \( P \), then mathematically:

\[ F = \frac{dP}{dt} \]

Sometimes for mathematical clarity we us the notation of \( P \) with a dot on top to denote the first derivative of \( P \), and with 2 dots for the second derivative. Thus, in this new notation:

\[ F = \frac{dP}{dt} = \dot{P} \]

You may also recall from the section on the rocket equation that momentum is the product of the mass and velocity. Thus we can say that the momentum of the flow from the nozzle of the rocket has a momentum:
Thus:

\[ F = \frac{dP}{dt} = \dot{P} = \frac{d(mv_e)}{dt} \]

If the exhaust velocity remains constant, which is a reasonable assumption, we arrive at the equation:

\[ F = v_e \frac{d(m)}{dt} = \dot{m}v_e \quad \text{...... equation 1} \]

The term “m-dot” is known as the mass flow rate, in other words the rate at which mass is ejected through the nozzle in kg/sec. In other words this is the rate at which the rocket burns fuel. This is an interesting relationship, which can be expressed in words as:

*Momentum Thrust = mass flow rate x exhaust velocity*

**Flow expansion**

The propellant is accelerated into the atmosphere. As it leaves the nozzle the propellant has an exit pressure \( P_{\text{exit}} \) and enters an atmosphere which has a pressure \( P_{\text{atm}} \). The transition from one pressure to the other cannot happen instantaneously as any pressure difference will cause a flow of high pressure fluid into the low pressure region. So what does this do to the thrust?

We define pressure as:

\[ \text{pressure} = \frac{\text{force}}{\text{area}} \]

So the force (a component of thrust called “pressure thrust”) depends on the pressure difference and the area of the nozzle. If the area of the nozzle is \( A \), we can produce an equation for the total thrust:

\[ F = \dot{m}v_e + A(P_{\text{exit}} - P_{\text{atm}}) \]
This suggests that we should aim for a maximum pressure in the nozzle so that the pressure thrust combines with the momentum thrust to produce a greater overall thrust. In fact, this intuitively correct result is wrong! If $P_{\text{exit}} > P_{\text{atm}}$ the exhaust gases will expand in all directions when they leave the nozzle, not only the direction of thrust. The total thrust of such a motor is less than could be delivered by just momentum thrust. We call this an “under expanded” flow as the propellant needs to expand more within the nozzle.

So what if $P_{\text{exit}} < P_{\text{atm}}$? In this circumstance the atmosphere will try to flow back into the nozzle. This causes sudden transitions from supersonic to subsonic flow to occur in the nozzle setting up shock waves. These shock waves turn some of the kinetic energy of the flow back into enthalpy, reducing the overall thrust. We call the flow “over expanded” as the flow expands too much in the nozzle reducing the overall pressure.

The ideal situation is when $P_{\text{exit}} = P_{\text{atm}}$ which only occurs over a narrow range of altitudes. This is not a major problem for modellers, as the burns tend to occur at low altitudes and over a relatively narrow range of atmospheric pressures. It is easy to design motors which are efficient over this range. It is a real problem for manufacturers of launch vehicles as the motor may burn from sea level to several tens of miles above sea level. It is normal practice on major launchers to tune the motor for an altitude around the middle of the range of pressures and accept some loss of efficiency at the start and end of the burn.

This effect is very pronounced on the Saturn V rocket. Next time you see any video of a launch, watch the plume. At launch it is long and thin as the flow is over expanded. At high altitudes the plume is very wide, exhibiting under expansion.
Propellant Grain

Solid propellants are the most common type used in model rocket motors. The propellant is ignited at the end away from the nozzle. The only escape route for the hot gas is to flow through the grain to the nozzle. As the gas flows through the grain it ignites all the exposed surfaces of the grain. As the surface burns away it exposes more grain to burn until it has all burned away.

The diagram shows a simple grain, a hollow cylinder. The area of the burning surface is the sum of the area of the top disc and the cylinder through the grain. As the burn progresses this surface area changes, thus the amount of hot gas changes. The amount of hot gas produced is directly proportional to the surface area.
The mass of hot gas produced per second is the mass flow rate, and thrust depends on mass flow rate. We saw earlier in this section that thrust is directly proportional to mass flow rate, so the thrust thus depends on the burning surface area. We can use this property to change the thrust profile.

By arranging the grain so that the burning surface area increases with time we get a profile where the thrust increases with time. This is called a progressive burn. Conversely if the area decreases with time we get a reduction in thrust or regressive burn. If the area stays constant we get constant thrust or a neutral burn.

In practice there is not a grain geometry which can give a truly neutral burn. Most neutral grains will give a degree of regression or progression.
Some common propellant grains used on model rocketry are shown below. Most black powder motors use an end burn. These are ignited from the bottom. Slotted tubes are used in medium and high power rocketry, and these are ignited from the top end of the motor.

**Specific Impulse**

How do we measure the effectiveness of a rocket motor? In cars we compare the effectiveness of motors through measures like miles per gallon of fuel, and time for 0-60 mph. The equivalent in rocket motors is called the specific impulse ($I_{sp}$). $I_{sp}$ is defined as:

$$I_{sp} = \frac{Ft}{W} \quad \text{(equation 2)}$$

Where $F$ is the thrust in Newtons, $t$ is the duration of the burn in seconds, and $W$ is the weight of fuel in Newtons. Overall this gives a measure of the impulse $Ft$ provided by a weight of fuel $W$. If we think about this, both $F$ and $W$ are forces, thus SI has the units of seconds. If we imagine rocket motor with an $I_{sp}$ of 300 seconds, then Newton of fuel (i.e. 1 kg under the acceleration due to earth’s gravity at sea level) will give 1 Newton of thrust for 300 seconds. The same amount of fuel could also give 150 Newtons of thrust for 2 seconds. It can be seen that the notion of $I_{sp}$ gives a measure of the effectiveness of a motor and fuel combination which is independent of the rate at which the fuel burns.

Some typical values of $I_{sp}$ are:

Black powder: 20-40 seconds  
Ammonium perchlorate and aluminium: 100-150 seconds  
Liquid oxygen and liquid hydrogen: 400 seconds.

By considering the mass flow rate of the motor as instantaneously constant, we can modify equation 1 to read:

$$F = \frac{m}{t} v_e$$

We also know that the weight of fuel $W$ is the mass of fuel multiplied by the acceleration due to gravity, so that
\[ W = mg_0 \]

Substituting for \( F \) and \( W \) in equation 2, and manipulating, we get:

\[ I_{sp} = \frac{v_e}{g_0} \quad \text{equation 3} \]

Thus specific impulse is directly proportional to the exhaust velocity, \( v_e \). The constant of proportionality is \( 1/g_0 \), where \( g_0 \) is the acceleration due to gravity at sea level.

Why is equation 3 significant? It shows that the higher the exhaust velocity the more efficient the motor becomes. In theory, we can keep on increasing the exhaust velocity and hence the efficiency of the motor. There are practical issues such as chamber temperature, pressure and flow expansion which limit the efficiency of chemical motors. Once outside the atmosphere we can accelerate ions to very high velocities in the vacuum of space, and thus get ion propulsion motors with \( I_{sp} \) of many thousands of seconds, but that is another story.....