Relativity and Rocketry

By Phil Charlesworth

In 1905 a little known patent clerk and part-time scientist published three scientific papers that laid the foundations of modern physics. The first paper explained Brownian motion, and led to the acceptance of the existence of atoms. The second paper explained the photoelectric effect and laid the foundations of quantum physics. The third paper was the theory of special relativity, and the patent clerk who wrote it was Albert Einstein.

Any one of these papers would have assured Einstein's fame. It was for his paper on Brownian motion that he received the Nobel Prize in 1921, not his better-known work on relativity. A century after it was published, relativity is widely regarded as either the province of a few geeky scientists, or a convenient way of moving the plot along in second-rate science fiction films. In fact relativity is a part of everyday life; it's just that the effects of relativity are so small we tend to ignore them.

This article introduces special relativity, explaining the theory by using examples based on rocketry. It starts by introducing the basic ideas of relativity, leading to the slightly disturbing conclusion that space and time are not quite as well behaved as we'd like them to be. Later on it considers just how strange space and time really are. It finishes by considering how Einstein arrived at his famous equation $E=mc^2$ and what the equation means.

Basic Relativity

We all understand the idea of velocity, or do we? When we launch a rocket we know that it leave the launch rail at velocity, burns out at a higher velocity, and coasts until apogee where it has no velocity. Einstein's basic question, which led him to investigate relativity, was to ask "velocity relative to what?". To the observer on the ground the rocket has one velocity, to an observer flying alongside the rocket, it appears to be standing still.

A rocket has a velocity relative to an observer standing on the surface of the Earth, but this is not the only possible reference point. The rocket has a different velocity relative to a car driving past the launch site, another velocity relative to the aircraft infringing our NOTAM, and many other possible velocities relative to other moving bodies such as the planets and stars. Einstein questioned whether there was an absolute reference in a universe where everything is in motion relative to everything else.

In the case of a rocket, it is convenient to measure the rocket's velocity relative to a point on the surface of the Earth. Consequently all people on the surface of the Earth who can see the rocket will see it travelling at the same velocity v. If we call the Earth our "frame of reference", then the rocket's velocity is measured *relative* to our frame of reference. We can turn this concept around by imagining that the rocket is the frame of reference. An observer on the rocket will see the rocket standing still while the Earth moves away at the same velocity v. Both of these frames of reference are equally valid.

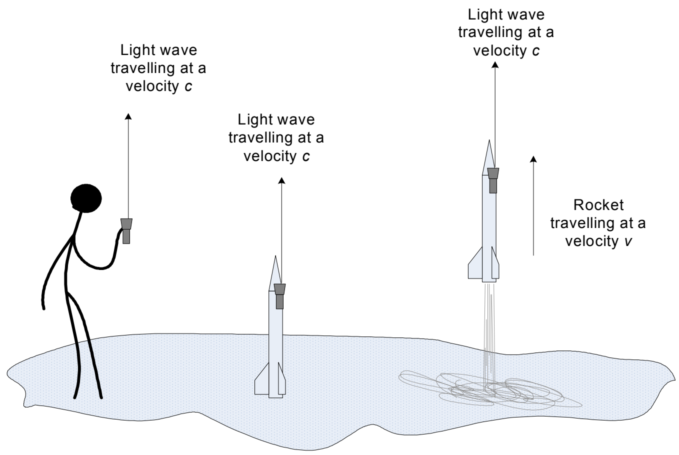
A two-stage rocket provides another consideration of relativity. It is reasonable to expect that the final velocity of the sustainer stage (relative to the Earth) is the sum of the velocities imparted by the booster and sustainer motors. In other words:

$v_{final} = v_{booster} + v_{sustainer}$.

If we consider the booster as our frame of reference, we find that $v_{final} = v_{sustainer}$. The final velocity, relative to any frame of reference, can be found by simply adding or subtracting velocities.

A “Thought Experiment”

Einstein liked to do “thought experiments”, which were experiments done entirely in the mind and without any equipment. In our “thought experiment” we’ll take two torches, one is held by a man on the ground and the other is attached to a rocket.



If the man shines his torch straight upwards, then the torchlight will go straight up at a velocity *c*, the velocity of light. This velocity can be measured relative to the Earth’s frame of reference. If we turn on the torch attached to the rocket then the lightwaves leaving that torch will also have a velocity *c* relative to our Earth frame of reference. We now launch the rocket and it flies at a velocity *v* relative to our Earth frame of reference. How fast is the light from the torch on the rocket travelling?

Relative to the rocket’s frame of reference the light is travelling at the velocity of light, *c*. As the rocket is travelling at a velocity *v* relative to the earth, then our man on the ground should “see” the lightwaves travelling at a velocity $v+c$ relative to the Earth’s frame of reference. Unfortunately the man sees all the lightwaves from both torches travelling at the velocity of light, *c*, relative to the Earth’s frame of reference.

So how can the velocity of light be the same in both the Earth’s and rocket’s frames of reference when one is moving relative to the other? According to the theory of special relativity the velocity of light is constant when observed from *any* frame of reference, so the man on the ground *must* see the light wave travelling at a velocity *c*. This is a rather strange and unexpected result that contradicts our previous ideas about adding and subtracting velocities.

Velocity, Distance and Time

Special relativity implies that all physical laws are the same in every frame of reference. Consequently, when one frame of reference moves with respect to another the velocity of light is identical in both frames of reference. We all learned at school that:

velocity = distance ÷ time

In Einstein's universe, the velocity of light is constant, so time and distance must be doing something strange. Special relativity shows that time and distance, which we rely upon to be constant in our measurement of velocity, depend on the frame of reference from which they are observed. Einstein suggested that the only reason that we don't notice this in everyday life is because we travel at such incredibly slow velocities, relative to the velocity of light, that we fail to notice the effects of relativity.

Because of special relativity all measurements of distance and time depend on the frame of reference in which they are measured. Experiments have shown this to be true, not only in the measurement of one-dimensional distance, but also in three-dimensional space.

Spacetime

It should be clear by now that our notions of space, which has 3 dimensions, and time, the fourth dimension, depend on the frame of reference from which they are observed. The universe in which we live is thus a strange place in which the four dimensions of space and time, generally referred to as "spacetime", don't behave properly.

One of the consequences of the theory of special relativity is that the amount by which spacetime "misbehaves" can be predicted. The amount of misbehaviour can be calculated using Lorentz's transformation, usually denoted by the Greek letter gamma γ. Gamma is defined by the equation:

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

We can use the value of gamma to calculate the values of length and time that we would observe when looking at other frames of reference. In this equation v is the velocity of the moving frame of reference, and c is the velocity of light (300,000,000 m/s).

We can do some more "thought experiments" in spacetime to illustrate the usefulness of gamma. These experiments led Einstein to the conclusion that energy and matter were interchangeable, and to some of the more interesting consequences of special relativity.

So What is Gamma?

Gamma allows us to take a measurement in the moving frame of reference and convert it into the value that the observer in the other frame of reference would measure. Putting this into a "word equation":

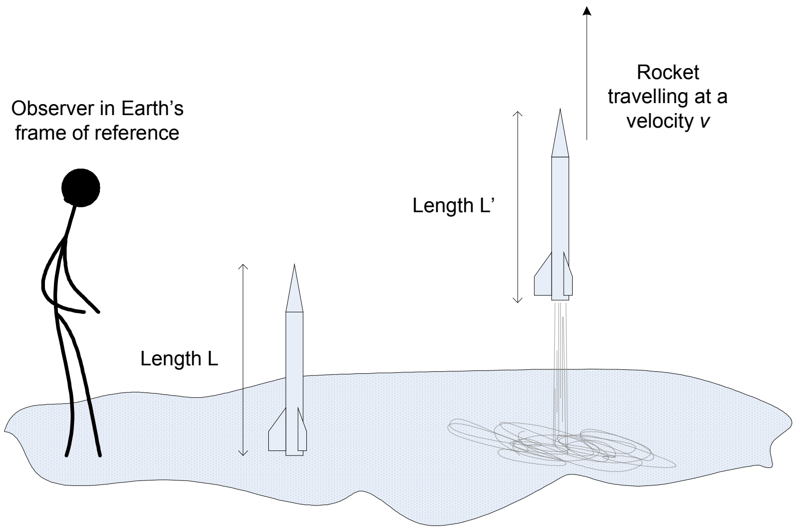
Length as seen by observer on Earth = length in rockets frame of reference $\div \gamma$

$L_E = L_R \div \gamma$

For example, if v is 90% of the speed of light ($v/c=0.9$) then we can calculate that $\gamma=2.3$. If we want to find out how big a moving rocket would be to an observer on Earth, then we simply divide the actual length of the rocket by 2.3. A pair of "thought experiments" best explains this slightly bizarre and counterintuitive idea.

Spatial Dilation

In the first "thought experiment" we'll consider a rocket being launched and observed by someone standing on the Earth. If we measure the length of the rocket before launch, i.e. while it is stationary in our frame of reference, and find it has a length L.



After we launch the rocket it is travelling at a velocity v in the Earth's frame of reference. An observer in the rocket's frame of reference would still measure the length of the rocket as L, whereas an observer in the Earth's frame of reference would measure the length as L_E. In the rocket's frame of reference the length of the rocket L_R is L. Consequently in the Earth's frame of reference we should see the rocket having a length $L_E = L_R \div \gamma = L \div \gamma$.

We call this change in length "*spatial dilation*" as the space occupied by the rocket seems to reduce, or dilate. How big is spatial dilation? If our rocket was travelling at a velocity $v=0.9c$, at which speed $\gamma=2.3$, then it would have a length L_R =2 metres in the rocket's frame of reference. To the observer in the Earth's frame of reference the rocket would only appear to be 2÷2.3=0.87m long. It appears to have shrunk to less than half its length.

If we consider velocities more likely to be encountered on the rocketry range, a 2 metre long rocket travelling at v=300m/s would appear to shrink by 0.000000001 mm. This is a very small amount, and we would not expect to notice it.

Time Dilation

The effects of relativity also impact on our measurement of time. In another of Einstein's "thought experiments", called the "twins paradox", one of a pair of twin brothers journeyed to another star system on a spacecraft at a velocity close to c. When the twin returned, he found that the brother who stayed behind had aged.

As with our previous thought experiment the two frames of reference are the Earth and the spacecraft. Just like spatial dilation, the amount of ageing depends on the relative velocities of the two frames of reference. If our planet was 10 light years away and the spacecraft travelled at v= 0.8c, then the trip will take 25 years in the Earth's frame of reference, but only 15 years in the spacecraft's. The twin who stayed behind will be 10 years older than his brother.

Why is this? The brother on Earth knows that the spaceship will travel at 0.8*c* relative to him and cover a round trip of 20 light years. The time that should elapse is thus 20/0.8 years, or 25 years. The brother on the spacecraft experiences a time interval of $25/\gamma$ years, which is only 15 years.

Mass Increase

Another unexpected effect of relativity is that the mass (inertia) of a rocket will increase as it accelerates. All mentions of mass in this article, so far, have been the mass at zero velocity, generally called the "rest mass". When a rocket moves, it behaves as if it has a mass $M=\gamma m$. As the velocity increases, the value of gamma increases and the rocket behaves as if it has more and more mass. As the rocket's speed approaches the speed of light its mass increases without limit until it has infinite mass.

I used this argument to try to convince one of the girls at work that she'd lose weight by standing still, but that's another story….

The Famous Equation

One other consequence of relativity is that our understanding of energy needs to be refined. Our definition of kinetic energy, $K=\frac{1}{2}mv^2$ only holds true at speeds which are very much less than the speed of light. A more complete equation for kinetic energy must take account of the frame of reference in which it has the velocity v:

$$K= \gamma\, mc^2 - mc^2$$

A bit of mathematical manipulation (a series expansion of $(1-x)^{-\frac{1}{2}}$ for those who enjoy maths) allows us to establish that the kinetic energy possessed by a rocket <u>approximates</u> to $K=\frac{1}{2}mv^2$ at velocities very much lower than c. Further manipulation shows that the total energy possessed by the rocket is:

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

When the rocket is stationary in out frame of reference, in other words v=0, we get the familiar equation:

$$E= mc^2$$

Why is this equation so important? It implies that the energy and mass of the rocket are interchangeable. They are different aspects of the same thing.

Consequences of Special Relativity

If mass and energy are interchangeable then accepted laws of physics such as the conservation of mass and conservation of energy are no longer true. If a small amount of mass ceases to exist, as happens in radioactive decay of an atom, then it must reappear in the universe as energy. Mass and energy are not conserved, but "mass-energy" is conserved.

Multiply mass by the square of the speed of light means that a little bit of mass is equivalent to a lot of energy. Every Kg of mass is the equivalent of 100,000,000,000,000,000 Joules of energy. To put this number into perspective, if a rocket weighing 1 kg sat on a table in the UKRA hut suddenly ceased to exist it would release enough energy to power 25,000,000,000 electric fires for one hour; and HSE worry about a few kg of AP….

Our fundamental ideas of time and distance, and everything we base on those ideas, need to be revised. Newtons Laws are clearly incomplete descriptions of the universe, as both space and time depend on the frame of reference from which they are observed. They are, at best, approximations that are only true at low velocities.

So to summarise: When a rocket moves it will shrink in size, get younger than the person launching it, and increase in mass. If it suddenly ceased to exist then the energy released would make an H-bomb look like an Estes ejection charge.

If by now you think the world of special relativity is strange, you should try Einstein's theory of General Relativity. Perhaps I'll wait until its centenary in 2016 before writing that article. Meanwhile, if you want to read more about Special Relativity, try "Relativity" by Albert Einstein, published by Routledge. Einstein wrote it in 1916 to introduce the ideas of special and general relativity to non-scientific audiences, and produced a book that can be read at many levels. At only £7.99 it's a great read.